The Arithmetic Teacher

APRIL · 1955

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THE ARITHMETIC TEACHER

Published Quarterly by
The National Council of Teachers of Mathematics

Editor: BEN A. SUELTZ, State University Teachers College, Cortland, N. Y.

Associate Editor: ESTHER J. SWENSON, University of Alabama, University, Ala.

Subscription Price: $1.50 per year (four issues) to individual subscribers, $2.50 per year to others (libraries, schools, colleges, etc.)

All editorial correspondence, including books for review, should be addressed to the Editor. Advertising correspondence, subscriptions to THE ARITHMETIC TEACHER, and notice of change of address should be sent to:

The National Council of Teachers of Mathematics
1201 Sixteenth Street, N.W., Washington, D.C.

Entered as second-class matter at the post office at Washington, D. C., June 21, 1954, under act of March 3, 1879. Additional entry granted at Menasha, Wisconsin.

The National Council has for its object the advancement of mathematics teaching at all levels of instruction. Any person who is interested in the field of mathematics is invited to apply for membership. Teachers should include the name and address of their school on their application. The membership fee is $3 per year and this entitles each member to receive *The Mathematics Teacher*, the official journal, monthly except in June, July, August, and September.

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A Philosophy of Arithmetic Instruction

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Needs

THE CONCEPT OF *need* pervades all modern theory with regard to learning. Parents and teachers are asked to discover and help the child satisfy his needs. The child's needs are to be at the center of all educational programs. However, there has been little or no suggestion that the child must also sense the need of understanding his parents and teachers, and of their guidance and counsel. The school has as its primary task not only to serve the child to satisfy his immediate needs, but also to equip him for service to society in his later life. Hence the child has need for that elementary mathematics which is demanded of all people in our free democratic society. Children will recognize some of these needs in their own daily experience, but other needs must be shown the pupils through experience supplied by the teacher.

In this connection it is well to note that in his experience alone a child will never meet all number situations called for in later life. Nor can he, from a large number of isolated arithmetic situations, ever come to have a basic knowledge for use in later life. Accordingly, we must teach the pupils a structure of arithmetic and sufficient applications of the structure, so that in new situations, in life problems, he can use the structure for the necessary solution of problems.

A need is a feeling for something which is absent, which if present, will *tend* to give satisfaction. These needs are biogenic, such as the need for food, shelter, survival, and so on. They are also social such as status, power, intellectual satisfaction, acceptance into society, and so on. It is these latter needs with which arithmetic learning is mostly concerned. The teacher can find these needs in a number of places. They are in our everyday cultural environment as indicated in the check list of the National Council of Teachers of Mathematics. The needs for intellectual, esthetic, and social activities may frequently arise simply and naturally from the nervous system and its interrelation with its environment. Not all children lack curiosity and not all children dislike arithmetic. The teacher is also well aware of the needs in the many, many social problems which the child will not encounter until he is much older, such as saving and spending the income, investment, insurance, loans, statistical interpretation, and so on. However, a beginning can be made by creating a make-believe adult situation for the child. (Playing store is one make-believe adult situation.)

Planning

In planning a program the teacher must at all times recall that arithmetic is a logical structure as well as a social instrument. Unless these two concepts are continually interrelated, we shall not succeed

in reaching our objective of an adult who *knows*, *recognizes*, and *uses* arithmetic in his daily life problems. While different parts of arithmetic can be learned in various orders, the logical order cannot be violated. We must learn some addition of whole numbers before we learn some meaningful multiplication. We must know the operations with whole numbers before we can learn to operate reasonably with fractions. But we can learn some of all the operations, one after the other, in some logical order, rather than learn all of one operation before we proceed to the next. To see the structure of arithmetic as a whole, rather than as separate parts, appears to give better success in its study.

Pupils sense needs for arithmetic in the world about them, at home, on the radio, in the papers, in their games, and so on. They seldom sense a need for a complete understanding and mastery of a number system. Real problems, and problems to which reality can be given, may be used to show the pupils the need of knowing arithmetic. The experiences, however, must not produce emotional reactions which detract from the learning of arithmetic; the experience must create within the pupil a drive which makes the learning of the arithmetic obligatory to him.

All of this suggests that we plan our program of instruction with our classes. Of course it is impossible for a group of children in grades 3 to 5 to plan its own program. Children are totally incapable of reproducing what great minds operating through centuries of time have created. We can, however, from time to time, discuss with the children what we are going to do, what they think they need, and how they can best learn what thus becomes their objective was well as the teacher's objectives. We can seize on all aspects of children's interest to motivate their learning. Instead of allowing outside interests to distract attention from arithmetic, a wise teacher uses them in promoting the study. Radio, television, the movies, the newspapers, even the comics are filled with arithmetic and a little search will reveal profitable examples. The resourceful teacher will adapt all these experiences to the level of maturity of his pupils. She will also look for causes of fear, defeat, and rejection among a few pupils and find ways to reassure and help them through their interests. However, in motivating the learning through interest in experience, we must take care that eventually it is the mastery and understanding of arithmetic that is achieved, as well as a fine personality.

Meaning

Granted the child sees the need for knowing how to multiply, how shall he learn it? Shall we show him how and drill, and expect that he will know? Shall we expect him to understand (have some meaning of) every particular algorism we use? The basic philosophy of meaning as a necessary concomitant of learning is now generally accepted. The degree to which meaning is necessary, how to obtain it, and what this does to drill, are still debated issues in arithmetic teaching.

A fact, concept, or operation is meaningful to a child when he relates it to his previous learning in such a manner that it becomes a working aspect of his behavior. How well it works depends upon the child's intelligence and the degree of practice he has in using the arithmetic. Drill (or practice) is the continued repetition of an act within limits of human variation. We should note that meanings may be correct or incorrect, and repetition of incorrect meanings or operations can interfere greatly with further learning. Suppose to a child a fraction has come to have the incorrect concept "2 out of 3 for $\frac{2}{3}$," "3 out of 5 for $\frac{3}{5}$," etc. Then if he is asked to add $\frac{2}{3}+\frac{3}{5}$, he says 5 out of 8 or $\frac{5}{8}$. Thus the whole thing has meaning to him, but not the meaning we desire. The more this child practices the more he is in trouble. In this case practice makes not perfect, but a perfect mess. So we must be very careful that the meaning the child puts into his arithmetic is the correct meaning.

Once a child has put meaning into an operation or a relationship, there must be practice or drill to secure facility and accuracy thereafter. This drill can be of a number of different types. The most common usage of drill is the repetition of the same isolated facts until it can be quickly recalled. This procedure usually depends on pure rote learning. An almost similar type of drill is the repetition of various isolated facts, such as 6×3, 8×7, 2×5, $7+8$, etc. This process of securing retention of concepts is also one closely related to rote learning. The more modern concept of drill is the repetition of varied facts in varied situations. This process follows the field psychology point of view; the seeing of the relations of the parts to the whole structure of arithmetic in as many aspects as possible. We believe that it will give a more permanent and more useful learning. When a new operation has been related to some past experience or concrete situation so as to be of use in new situations, it has become meaningful. Depending on how it is learned the multiplication table can become a mere rote mechanism soon to be forgotten, or it can become a long-remembered meaningful relation of numbers in an arithmetic operation.

Practice

Once children have sensed the mathematical operation, have put meaning into it, and see where it is of value, they are ready to drill. Then and then only will drill be effective. Buckingham[1] has pointed out that drill is effective only when to the child it has *purpose*, he senses its *value*, he has *confidence* in his ability to perform because of some familiarity with the operation, so that he can *direct* himself toward the deepening and broadening of his understanding.

John Dewey has expressed the place of drill in arithmetic, by two excellent statements. "Practice skills can be intelligently, non-mechanistically *used*, only when intelligence (meaning) has played a part in their learning" and "An erroneous conception widely held is that since traditional education rested upon a concept of organization of knowledge that was almost completely contemptuous of living (present experiences), therefore education based upon living experiences should be contemptuous of the organization of knowledge."

What then is the place of drill? It is present and must be present in our learning. The arithmetic used for solving quantitative problems demands *facility in computation which is obtained only through practice.* The learning of division and percent, and other applications in later grades, demands facility in the use of the more elementary operations, so that the mind can give its attention to the newer concepts and relations. This faculty is gained only through practice. However, the practice comes after meaning; it comes in varied situations and problems, and because of meaning the learning is faster, more permanent, and less practice is needed than is necessary for rote learning.

Learning

The teacher may now well ask, how do we get the child to put meaning into his arithmetic? We can seek an answer to this question in asking our selves how *we* do it. A newspaper article said the U. N. forces advanced 6 miles all along a 60 mile front. What meaning do you see in this? Do you see a soldier walking forward 6 miles, and others to his left and right doing this also? Do you see $6\times60=360$ square miles of territory has been covered? However or whatever you do, you usually go back to concrete experience and that is where learning of all type begins. So multiplication, division, fractions and so on, always have the initial learning phases embedded in concrete situations. All learning begins in a concrete experiential problematic situation in which the organism is motivated to find a solution.

[1] Burdette R. Buckingham, "What Becomes of Drill," *Arithmetic in General Education*, pp. 196–225. Sixteenth Yearbook of the National Council of Teachers of Mathematics, 1941.

Motivation is goal-directed behavior. The motivation is entirely within the child, but it is brought about partly by the environment acting on the child, and partly by the child's own internal structure. These factors cause nervous tension of the energies within the body, and the release of these tensions in physical and mental activity is the drive that sends the pupil toward a solution, that is, to learn. Without this motivation and drive, no real learning takes place. So we try to develop the type of environment, the type of stimuli that help the child to motivate his learning. When this motivation is present, then self-instruction (and all real learning is ultimately self-instruction) takes place. This motivation has its first seeds in experience, concrete sensual experience. Here is where meaning begins. But it does not and should not end at this point. However, the uses of visual and mechanical aids for learning and re-learning through experimentation is a first step toward developing meaning.

The teacher must next encourage the child to abstract certain recurring properties in his experimentation and to generalize these relations into arithmetic rules and principles. These general principles must also be related to previous generalizations. Thus, the proper placement of partial products in multiplication must be related to previously learned generalizations regarding multiplying by 1, 10, 100 etc. The adding of these partial products to get the total product must also be related to the general principle of distribution.[2] It is such generalizations that the meaning of multiplication deepens and the relationship of one operation to another takes on the aspect of a structure. This is not explicitly recognized by pupils in the beginning, but the implications will later develop into a broader understanding of the subject. It is quite evident that such learning demands more, and more carefully planned instruction on the part of the teacher. The learning may proceed more slowly, but in the end we shall get a better educational product.

Under such learning, there is a need for continuous evaluation of the patterns of thinking going on in the child's mind. Computation tests with pencil and paper are not sufficient for this. Frequent oral explanations by the pupil form a better testing device for evaluating understandings. But more important, from a point of view of learning, the children must by their own planning and teacher guidance, constantly measure their own progress, come to recognize their own weaknesses, as well as strength, and by the use of teacher-prepared tests discover where they stand, what they need, and seek advice on how to get it. We must have children more and more take on *responsibility* for their own progress in learning. This is a long-neglected aim in all school instruction.

Problem Solving

Along with this generalization and structuring of the knowledge of arithmetic, we must plan for socializing our instruction, that is for the use of arithmetic in problem solving. In good instruction, problem solving is always present for problem solving is learning and learning is problem solving. To develop the ability to solve new problems by the use of arithmetic is the greatest objective of our instruction. It is in problem situations that arithmetic can best be seen as a whole—as a completed body of knowledge. At this point, many teachers will no doubt ask: "How do you teach problem solving?"

This is in large measure an unanswered question, but many valid suggestions to aid the developing of problem-solving ability are at hand. Perhaps most significant are (1) always look at the whole problem, the whole situation; (2) seek the relationship of the parts to the whole, and the whole to the part; (3) analyze, or-

[2] The distributive law is fundamental to all mathematics and is given symbolically by $a(b+c)=ab+ac$. Hence, 256×68 is $256\ (60+8)$ or $60\times 256+8\times 256$.

ganize and reorganize the relationships until what is known is directly related to what is wanted, then insight will occur. Thus problem solving demands "*ceaseless attention to the building of clear, well inter-related arithmetic concepts in all the areas of common experience.*"

Preliminary to the solution of word problems are three characteristics of arithmetic instruction that have received all-too-little attention in the past. They all rely on active thinking as a mode to learning rather than on passive listening to instructions. The first is *estimation*. Of course, to make intelligent estimates one must have rather clear cut concepts of number, the number system, and the operations. To find the cost of three articles each priced at 48 cents, the second-grade child (since he knows the number system) says this price is a little less than 3 half dollars or \$1.50. As he advances through the grades he continues this estimation in far more difficult situations. The second characteristic is *mental solutions* to both computations and problems. Using the knowledge he has of the structure of arithmetic, here the child reasons without the aid of pencil or paper. He does not do mental gymnastics or visualized computations, but he does secure the accurate answer. In the preceding problem he reasons each article is 2 cents less than a half dollar—3 times 2 is 6—the price is 6 cents less than \$1.50 or \$1.45 minus 1 cent or \$1.44. This type of mental solution is to be practiced from grade 1 on up into high school, the problems increasing in complexity each year. Estimation and mental solutions are the great practices that business men use every day.

The third characteristic is the association of language with an *operation*. As subtraction is taught, such language as—*take away,—how much more—what is the difference—what is left—how much less—minus—less than—lost*—and so on are used in connection with concrete objects, so that the child associates a manner of speaking with the operation of subtraction. In this way, he is able to recognize a phrase or sentence in a word problem as indicating the separating of a group from a larger group to the size of the remaining group, or the comparing two groups of unequal sizes to find their difference. Language and concepts are highly interdependent in arithmetic.

The foregoing philosophy of instruction can and should lead to an educational product that has a healthy attitude toward arithmetic. It should result in a person who has self-assurance in the field of arithmetic because of a genuine understanding of the mathematical implications as well as the ability and compunction to recognize and use arithmetic in all life's quantitative situations. What psychology of learning is this philosophy based upon? Upon all those principles which have been common and supported by research findings in the several psychologies. The psychology largely used is that of the gestalt school, but is is supported by connectionism under its modern interpretation. To go further here into any of these psychological theories would be of no great help to interpret the point of view expressed.

Summary

Learning arithmetic stems from the needs for arithmetic as perceived by the learner. The child and the teacher together discuss and plan these needs. Arithmetic can be learned only if it has meaning to the child. Meanings in arithmetic arise from thinking about things, from concrete experience, and from problem situations. However, meanings and experience are only the beginning. They must be organized into some sequential structure, and the facts must be made nearly automatic (that is easy to recall) through practice. Thus meaning and understanding—that is, concept building—precede practice or drill. The child learns to evaluate his own progress with the help and guidance of the teacher. All testing is for the purpose of learning and motivating learning. The final goal of all instruction

is to develop within the mind of each child a problem solving ability in quantitative situations. This ability is best acquired through a problem solving approach to learning arithmetic operations as well as by practice in real problem situations.

EDITOR'S NOTE: Professor Fehr has applied some of the modern principles of learning to the field of arithmetic. The article warrants careful reading. Mr. Fehr calls attention to the role of the child in learning, to his responsibility as a learner, and to various modes of achieving learning. Teachers and supervisors should be interested in the positive statements about such things as children planning their own programs and the role of drill or practice. Professor Fehr has drawn his principles of learning from several psychologies. He would probably agree that we still know much too little about how the minds of different children operate.

PUZZLERS

A BOOKWORM BEGINS at page one, volume one of a ten-volume set and bores straight through the pages and covers of the set to the last page of the tenth volume. If each cover is one-eighth inch thick and each volume has two inches of pages, how long is the hole bored? Assume that the books are arranged in sequence from left to right on a shelf. Did you get the right answer the first time? Check with a series of books on a shelf.

GEORGE FOUND A KNIFE that cost $1.00 when new but was worth 50 cents when he found it. He sold it for 25 cents. What was his per cent of profit on the investment? How can the answer be expressed?

AN IMPORTANT ANNOUNCEMENT

NCTM Membership to Be Given to "Arithmetic Teacher" Subscribers

Beginning with the 1955–56 school year, subscribers to the ARITHMETIC TEACHER will be given membership privileges in the National Council of Teachers of Mathematics and the frequency of publication will be increased from four to six issues per year. The change, which has been made in response to a wide demand, will make sub scriptions to this journal of even more value than in the past.

The new plan calls for a uniform membership fee of $3.00. This fee will cover full membership privileges and benefits, including the rights to vote and hold office, and a subscription to either the *Mathematics Teacher* or the ARITHMETIC TEACHER. Those who wish to receive both journals may have them for the special price of $5.00, provided both journals are sent to the same address and have the same expiration date.

The student membership fee, which is available to any student who has never taught, will remain at $1.50, with student members also having a choice of journals. Student members who wish to receive both journals may obtain them for $2.50, provided both journals are sent to the same address and have the same expiration date. (Student membership does not include the rights to vote and hold office.)

The change in fee will go into effect on ARITHMETIC TEACHER subscriptions or renewals for which mailing begins with the October 1955 issue. Persons who wish to receive both journals at the reduced joint rate should arrange to have their subscriptions run for the same period of time. In cases in which a person now receives both journals with different expiration dates, a short-term subscription will be accepted in order to make the dates coincide.

The rates to institutions (schools, libraries, departments, etc.) will be $5.00 for each journal. Common expiration dates are not necessary for institutions that subscribe to both journals. An additional mailing charge of 25¢ to Canada and 50¢ to foreign countries is required for either journal on both individual and institutional subscriptions.

Requests for further information should be addressed to M. H. Ahrendt, Executive Secretary, National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D. C.

Development of Ability to Estimate and to Compute Mentally

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At the present time in all arithmetic programs increased emphasis is being placed upon development of ability to compute mentally and to obtain reasonably close approximate answers for computations in all four processes. Attention to this aspect of arithmetic teaching is essential because it contributes to both of the commonly accepted large general aims in the teaching of arithmetic: the social aim and the mathematical aim.

The Social Aim

Inherent in the social aim is the requirement that children see the practical value of the arithmetic they are learning. They need to realize that the better they understand numbers, the more skill they possess in the operations of arithmetic, and the oftener they put these understandings and skills to productive use, the more adequately will they be able to meet the steadily increasing quantitative demands of daily living. In many types of social experiences on both the child level and the adult level, the need to obtain answers mentally arises more frequently than the need to compute with paper and pencil. Often an estimated answer obtained by mental computation with rounded numbers provides an adequate basis for a decision when the number relationships involved are such that an exact answer can not be found mentally. Even when exact answers are required in life situations, it is important that both children and adults recognize the advisability of utilizing an approximate answer as a check upon the reasonableness of the exact answer they may obtain.

The Mathematical Aim

The mathematical aim maintains that pupils need to gain an ever-expanding understanding of the structure and organization of the number system if they are to develop a high degree of competence in the procedures of arithmetic. Mental computation and estimation not only provide opportunities for the utilization of meanings pupils have acquired, but also stimulate the development of more mature understandings of basic principles and number relationships. As we shall see in the illustrations which follow, pupils who succeed in mental computation and in estimating do not always employ standardized, prescribed thought patterns. Instead, these pupils develop *ingenuity* and *resourcefulness* in dealing with numbers.

Although the importance of developing ability to estimate and to compute mentally is recognized, many teachers are concerned because of the difficulties they encounter in trying to attain these learning outcomes. This is particularly true in Grade 6 during the teaching of multiplication and division of mixed numbers and decimals. Pupils seem to have become so accustomed to finding an exact answer immediately that they are not disposed to take time *first* to analyze the number relationships involved, use round numbers, and obtain an approximate answer to give some indication as to what the exact answer should be. Also many pupils have become proficient in carrying through computational procedures in a mechanical fashion without understanding why the procedures give correct answers. These

children resist the introduction of ways of working and thinking that differ from the ones they have habituated.

There are, however, in nearly all classes some pupils who have developed the ability to estimate and to compute mentally. We may well explore the types of situations in which estimation and mental computation are desirable and consider the steps which teachers may take to achieve this very important goal of arithmetic teaching. Let us listen in, as pupils on different grade levels explain their thinking. It will be noted that some pupils find exact answers mentally by adjusting the estimated answer. Some pupils find it necessary to record partial answers as they estimate.

Ways of Thinking in Estimating

Mary, a 3rd grade pupil, is explaining how she estimates the sum for Ex. 1. "28 is about 3 tens or 30; 33 is a little more than 3 tens; and 18 is nearly 2 tens, or 20. Then I think 30, 60, 80."
"I like to write the tens I add when I estimate."

Tom, in the same class as Mary, just looked at the numbers in Ex. 1 and said: "30, 60, 80. The sum is about 80."

(1)

$$\begin{array}{rr} 28 & 30 \\ 33 & 30 \\ +18 & +20 \\ \hline 79 & 80 \end{array}$$

Peter is estimating the answer for Ex. 2.
"91 is about 90; 38 is about 40; 40 from 90 leaves 50."

Janice thinks the exact answer for Ex. 2. by adjuting her estimate.
"91 less 40 = 51. The real answer is 2 bigger, or 53."

(2)

$$\begin{array}{r} 91 \\ -38 \\ \hline 53 \end{array}$$

In Grade 4 we find that pupils often use a grater variety of ways of estimating or computing mentally. Consider the different ways of thinking used by these children for Ex. 3.

Helen: "189 is about 2 hundred. 97 is about 1 hundred. 200 + 100 = 300, but I know that the exact answer is about 10 less, so my estimate is 290."

Jean: "189 + 100 = 289. The true answer is 3 smaller, or 286."

Fred: "18 tens + 9 tens = 27 tens, or 270; 270 + 9 = 279 and 7 more makes 286."

(3)

$$\begin{array}{r} 189 \\ +97 \\ \hline 286 \end{array}$$

For Ex. 4, one child finds an estimate and two children are able to think the exact product.

Helen: "95 is close to 100. Then 6 × 100 = 600."

Betty: "600 is a good estimate. The exact answer is 6 × 5 or 30 smaller. 600 less 30 = 570."

Jim: "6 × 540; 6 × 5 = 30; 540 + 30 = 570."

(4)

$$\begin{array}{r} 95 \\ \times 6 \\ \hline 570 \end{array}$$

Some pupils generalize, as Henry did below in Ex. 5, that estimates in subtraction are closer if only the subtrahend is rounded.

Allen: "7320 is about 7 thousand; 1954 is about 2 thousand.
"2 from 7 leaves 5—the estimate is 5000."

Henry: "7320 less 2000 leaves 5320. This is about 50 smaller than the exact answer. 5320 + 50 = 5370."

(5)

$$\begin{array}{r} 7320 \\ -1954 \\ \hline 5366 \end{array}$$

Pupils find that rounding the dividend to the nearest hundred or thousand is sometimes helpful, but often it is not.

Linda said: "I paid $2.94 for a box of 6 handkerchiefs. About how much did one cost?"

Marion estimated: "2.94 is about $3.00. 1/6 of $3.00 = 50¢.
"You paid about 50¢, just a little less than 50¢."

(6)

$$\begin{array}{r} \$.49 \\ 6\overline{)\$2.94} \end{array}$$

For finding 1/7 of 595, Harry said: "7 × 8 = 56, so 7 × 80 = 560; 7 × 90 = 630. The number 595 is about halfway between 560 and 630, so the answer is about halfway between 80 and 90. My estimate is 85."

(7)

$\frac{1}{7}$ of 595 = 85

In Grade 5, the whole numbers in examples become larger and estimates are not likely to be as close as in Grades 3 and 4 where smaller numbers are involved. However, using round numbers and estimating will establish upper or lower limits for answers which prove helpful. Also, in Grades 5 and 6 pupils often write the round numbers which they use in obtaining estimates.

For Ex. 8 Jack rounds both factors.
Jack: "28 is about 3 tens; 39 is about 4 tens.
"3 tens ×4 tens = 12 hundreds (1200). The exact product is less than 1200 because I rounded both 28 and 39 upward."

George rounded only one factor in Ex. 8 to obtain a closer approximation.
George: "I multiply 28 by 40 to get an estimate.
40×20 = 800; 40×8 = 320; 800+320 = 1120."
George's estimate is closer to the exact product than Jack's but he found it necessary to write the steps in his thinking.

(8)

$$\begin{array}{r} 28 \\ \times 39 \\ \hline 252 \\ 84 \\ \hline 1092 \end{array} \qquad \begin{array}{r} 30 \\ \times 40 \\ \hline 1200 \end{array}$$

$$\begin{array}{r} 40\times 20 = 800 \\ 40\times 8 = 320 \\ \hline 1120 \end{array}$$

To obtain an estimate for Ex. 9, Carl finds an upper and a lower limit for the product and then judges about what the best estimate is.

Carl: "$5.48 is about halfway between $5.00 and $6.00
"19 is almost 20, 20×$5.00 = $100; 20×$6.00 = $120. Probably the product is between $100 and $120. My estimate is $110."

David finds a closer estimate by thinking of $5.48 as 5½, multiplying 19 by 5½.

David: "$5.48 is about $5½. 5×19 = 95. ½×19 is about 10. Then 95+10 = 105. My estimate is $105."

(9)

$$\begin{array}{r} \$5.48 \\ \times 19 \\ \hline 4932 \\ 548 \\ \hline \$104.12 \end{array}$$

In Ex. 10, Katie finds an upper and a lower limit for a quotient.

Katie: "176 is the first partial dividend. 176 means tens, so the quotient will be a 2-place number."
"Next, I think 30's in 170 or 3's in 17 are 5, so the quotient will probably not be smaller than 50, and it can not be as large as 60."

Bill: "For 1768÷34, I think 34×100 = 3400. 1768 is about half of 3400, so the quotient will be about ½ of 100 or 50."

(10)

$$\begin{array}{r} 52 \\ 34\overline{)1768} \\ 170 \\ \hline 68 \\ 68 \\ \hline \end{array}$$

Although there are relatively few applications of multiplication and division of mixed numbers in pupils' out-of-school experiences, all courses of study include these topics. There are so many steps involved in the procedures for finding exact answers, and an error in any step may result in an absurd answer. Many children are never quite certain which number to invert in dividing by a fraction or a mixed number. In studying these processes, all pupils will profit greatly by establishing the habit of obtaining an estimated answer to serve as a guide in judging the reasonableness of the exact answer.

Sam, in Ex. 11, thinks:
"8¾ is about 9; 2⅞ is about 3.
"9÷3 = 3. My estimate is 3.
"My exact answer is $3\frac{1}{23}$, which is a sensible answer when compared with my estimate."

(11)

$$8\tfrac{3}{4} \div 2\tfrac{7}{8} =$$

$$\frac{35}{4} \div \frac{23}{8} =$$

$$\frac{35}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{23} = \frac{70}{23} = 3\,\frac{1}{23}$$

Roger, in Ex. 12, explains:
"Rounding both mixed numbers, downward, the product would be 2×4, or 8. Rounding upward, the product would be 3×5, or 15.
"My estimate is 10, a number between 8 and 15. My exact answer, $11\frac{23}{48}$, is reasonable."

(12)

$$2\tfrac{3}{8} \times 4\tfrac{5}{6} =$$

$$\frac{19}{8} \times \frac{29}{6} = \frac{551}{48} = 11\,\frac{23}{48}$$

Kathy, in Ex. 13, utilizes her knowledge of fraction and decimal equivalents to help her to estimate a quotient with a decimal fraction divisor. She thinks:
".24 is close to .25, which equals $\frac{1}{4}$. 7.92 is close to 8.
"In 1 whole there are four $\frac{1}{4}$'s, so in 8 wholes there are 8×4, or 32 fourths.
"$8\div\frac{1}{4}=32$. My exact answer, 33, must be correct."

(13)

$$.24\overline{)7.92}=24\overline{)792}\quad\begin{array}{r}33\\ 24\overline{)792}\\ 72\\ \hline 72\\ 72\\ \hline\end{array}$$

In Grades 6 to 8, pupils find it advantageous to estimate answers in working with decimal fractions and mixed decimals. Pupils who understand decimals and place value readily apply skill developed in estimating with whole numbers to mixed decimals.

Janet, in Ex. 14, says:
"15.190 is about 15; 8.754 is about 9.
"9 from 15 leaves 6."

(14)

$$\begin{array}{r}15.190\\ -8.754\\ \hline 6.436\end{array}$$

Alfred, in Ex. 15, explains:
"2.125 is just a little more than 2. 64×2 or $2\times64=128$. The exact answer is larger than 128 because 2.125 was rounded downward."

John, in Ex. 15, thinks the exact answer:
"$2.125=2\frac{1}{8}$. $2\times64=128$. $\frac{1}{8}\times64=8$. $128+8=136$.
"The exact answer is 136."

(15)

$$\begin{array}{r}2.125\\ \times64\\ \hline 8500\\ 12750\\ \hline 136.000\end{array}$$

Ex. 16
Alice: "39.6 is about 40. 2.8 is about 3. $3\times40=120$.
"This is larger than the exact answer."

Harry: "I round 39.6 to 40. 2.8 is more than $2\frac{1}{2}$ but less than 3. $2\times40=80$. $2\frac{1}{2}\times40=100$. $3\times40=120$. The exact answer is probably about halfway between 100 and 120. My estimate is 110."

(16)

$$\begin{array}{r}39.6\\ \times2.8\\ \hline 3168\\ 792\\ \hline 110.88\end{array}\qquad\begin{array}{l}2\times40=80\\ 2\frac{1}{2}\times40=100\\ 3\times40=120\end{array}$$

Guiding Pupils in Estimating

What understandings, abilities, and values should we emphasize in our teaching of arithmetic to guide pupils to develop competence in estimating and in obtaining exact answers mentally? Some of these are discussed below.

1. *Pupils need to recognize the relative importance of approximate answers and exact answers in the quantitative situations of daily life.*

The teacher will need to be alert to the situations in which his pupils find socially significant uses for arithmetic. From these he will call special attention to those in which approximate answers and mental computation were required.

Pupils may be encouraged to discuss with their parents similar types of situations. The teacher will find many textbook problems suggestive and appropriate. As practical uses of estimation occur, these uses should be listed and kept throughout the year. Pupils in the upper grades who become genuinely interested in searching for occasions when approximation is needed will tend to find an increase in the number of uses in their own daily life experiences.

John (in the fourth grade) gave this illustration of a situation in which he estimated: "I wanted to pay for movie tickets for two friends and for myself. The tickets were 49¢ each. I had two dollar bills and a half dollar in my purse. I gave the ticket seller a dollar bill and the half dollar because I knew that 3 times 50¢ was $1.50."

Sally (in the fifth grade) told the class that she estimated in this situation. Her family planned to a trip of bout 1000 miles. She asked her father how many days it would take them. Her father told her that on trips they usually averaged about 40 miles and hour and planned to drive about 8 hours a day. Sally thought $8\times40=$

320 miles in 1 day; 640 miles 2 days; 960 miles in 3 days. She decided that 960 was so close to 1000 that they would probably take 3 days for the trip.

2. *Pupils need to gain an increasingly more mature understanding of the nature and structure of the number system.*

Pupils gain their basic understandings of numbers by working with objects which can be combined, separated, and grouped according to the standard sized group in our number system—the ten group. Pupils learn our system of notation, our way of keeping a record of the objects we have counted and grouped into tens and ones; into tens of tens, and so on. By gradual steps, pupils learn that the position in which a digit is written determines its value in the number, and that each position or place in a number has a value ten times the one immediately at its right.

By Grade 5, pupils have had experiences in building, analyzing, reading, writing, and using whole numbers through millions. They are then ready to extend their number meanings to include decimal fractions which represent merely the extension of the number system to the right of ones' place.

Many of the difficulties which pupils experience in working with decimal fractions are due to the fact that they have an inadequate understanding of the logic of the whole number system. If pupils are to become competent in obtaining either exact answers or approximate answers in the operations with decimal fractions, they need a wealth of experiences to make possible the development of these basic relationships of the number system.

While decimal fractions employ the same system of notation as whole numbers and a study of one should tend to clarify and extend understandings of the other, this is not the case for common fractions. To gain functional concepts of common fractions, pupils need many and varied experiences in manipulating fractional parts of real and representative objects.

3. *Pupils need to learn to round numbers and to develop judgment and resourcefulness in using round numbers to obtain reasonable approximations.*

As pupils in Grades 3 and 4 learn about place value of numbers and have practice in analyzing and building 3 and 4 place numbers in different ways, they should also learn the principles for rounding numbers to the nearest ten hundred, and thousand. Pupils should become familiar with situations in which round numbers are more meaningful and more useful than exact numbers. For example, it is probably just as accurate to think of the population of a given city as 256,000 as to say that it is 256,789. The rounded number can often be remembered, whereas the other one would be forgotten.

In Grades 5 and 6, as pupils increase their understanding of the number system, they should learn to use larger rounded whole numbers. When decimal fractions are studied, attention must be focused upon rounding the part of the number to the right of ones' place to the nearest, tenth, hundredth, thousandth, etc. In building meanings for mixed numbers, the relative significance of the whole number and of the added fraction needs to be clearly appreciated by pupils. Rounding a mixed number to the nearest whole number is important as pupils find approximate answers in multiplying and dividing mixed numbers.

4. *To develop competence in estimating, pupils need a thorough understanding of process meanings and their interrelationships.*

Pupils need to realize that the processes of addition, subtraction, multiplication, and division are merely ways to combine or separate groups. They need to know the basic principles which govern the way we combine groups in addition and in multiplication, and the way we separate groups in subtraction and in division.

In addition and subtraction of 2 and 3 place numbers, pupils find that the numbers are analyzed in terms of the ten-system, and that only like units may be

added or subtracted. To add $37+56$, we may find partial sums and then combine them to find the final sum. $30+50=80$; $7+6=13$; $80+13=93$. Or we may add the ones ($7+6=13$), regroup 10 ones as 1 ten, and add it with the given tens to obtain 9 tens and 3 ones, or 93. Pupils need to relate subtraction to addition and to understand that in subtraction we know the sum of two addends and one addend. To find the missing addend, we subtract.

In multiplication and division, likewise, we regroup number in terms of the decimal system. To find 3×58, we may find partial products and combine them to find the total product. $3\times 50=150$; $3\times 8=24$; $150+24=174$. Or we may find the product for the ones ($3\times 8=24$) and regroup 20 ones as 2 tens and add 2 tens to 15 tens, the product of the tens, to obtain 17 tens and 4 ones, or 174. The interrelationship of multiplication and division must be clearly discerned. We divide to find a missing factor when we know a product and one of its two factors.

To achieve success in estimating, pupils need to understand and to utilize basic relationships among the numbers involved in the computation to be carried out:

a) If the multiplier is larger than 1, the product is larger than the multiplicand. If the multiplier is smaller than 1, the product is smaller than the multiplicand.
b) If the divisor is larger than 1, the quotient is smaller than the dividend. If the divisor is smaller than 1, the quotient is larger than the dividend.
c) The quotient is not changed when both the dividend and the advisor are multiplied or divided by the same number.

5. *Pupils need many opportunities to practice estimating and to develop confidence in their ability to estimate.*

If pupils are to develop competence in estimating and in computing mentally, they must have carefully planned activities in a sequence that leads from work with simple examples in which number relationships can easily be discerned to more complex examples. Much of the work will need to be done orally and this is time consuming, but is worth the time it takes. The ways of thinking and working in estimating are not set, and pupils need to be encouraged to find and use a variety of number relationships. In their oral discussion, pupils share their ways of thinking with each other and take pride in their individual ways of working.

It is often advantageous for the teacher to work with small groups, grouping together pupils who can follow the reasoning of others in the group. More capable pupils in a class may be expected to make most of their computations with round numbers mentally. Pupils of average ability may be encouraged to record some of their work with rounded numbers. Permitting pupils to write computations with round numbers is comparable to permitting some pupils to show changed minuends in subtracting or showing carried numbers in adding columns. Teachers sometimes become discouraged in teaching estimating when they expect all computations with rounded numbers to be done mentally. This is a goal to work toward, but it must be approached gradually.

Whether a pupil thinks aloud before the entire class, before a small group, or for the teacher only, he should not be hurried as he tries to explain some relationship he has discovered. Pupils need praise and encouragement for each forward step. When it is evident that a pupil is on the wrong track in his thinking, he must be tactfully redirected. Teachers need to make certain that pupils do not think that estimating means guessing.

Summary

Pupils who have developed competence in estimating and in computing mentally find many opportunities to utilize these abilities in life situations. We can justify emphasis upon these learning outcomes in the teaching of arithmetic because they contribute to the development of intelligent mastery of the procedures used to

find exact answers. Pupils who have been taught to imitate and to habituate process patterns without regard to the meanings and relationships involved are not critical of the exact answers they obtain. A background of meanings and understandings is essential for development of the ability to estimate, but practice in estimating in turn fosters the building of more mature understandings, and promotes the development of ingenuity and resourcefulness in meeting all types of quantitative situations. When pupils develop the habit of analyzing the number relationships involved and of thinking out the answer to be anticipated, they are far more critical as they carry out the work to obtain the exact answer.

The program of mental arithmetic described in this article differs markedly from the mental arithmetic of an earlier day in several respects. First, it is a part and parcel of the regular program of arithmetic instruction, not something separate and apart. It would seem safe to assert that to teach arithmetic with meaning and understanding, learning activities of the kind used as illustrations in this article must be experienced by the learner. Second, the present day mental arithmetic requires thinking in terms of number relationships rather than the performance of the operations of arithmetic mentally by paper and pencil methods. Obviously the mental gymnastics of the older mental arithmetic could contribute little to greater meaning and understanding. The evidence gathered to date warrants the final conclusion that the current conception of mental arithmetic is more apt to produce skill in mental computation than the older program.

Editor's Note: Some may ask, "Why waste time on estimating when a direct method of computation will give the right answer in less time?" Miss Sauble has pointed out that estimated answers are often more like "thinking answers" and that frequently a precise answer is not needed or desired. The editor recalls the clerk in the store who insisted on giving him a dollar too much in change and showed him how she had taken the time to "do it with paper and pencil." Good methods of estimating are valuable in following a discussion or presentation of another person because they enable the listener to think and arrive at a reasonable conclusion, even to check the reasonableness of the other person's thinking with figures. We must recognize that the mental method usually proceeds by different steps, perhaps even different or reverse routes, from the written method. Query, is there an essential difference in modes of estimating that bright children employ from those which children of lesser ability can or should use? Do children of lesser ability in grade six tend to use the same methods that brighter children used in grade four? Does learning to estimate by applying a mechanical rule have any real value in the elementary school?

DR. CLARK BECOMES AN ASSOCIATE EDITOR

Dr. John R. Clark has been appointed as an additional associate editor of The Arithmetic Teacher. Dr. Clark is well known to people in mathematics education throughout the country for his writing of books and journal articles, for having edited *The Mathematics Teacher*, for lecturing at many teacher's meetings, and for his many years on the staff at Teachers College, Columbia University.

In a little more than one year The Arithmetic Teacher has grown from zero to nearly 5000 subscribers. During this period Dr. Ben A. Sueltz has served as editor and Dr. Esther Swenson has served as associate editor. Beginning in October, six issues will be printed each year. Mr. Sueltz and Miss Swenson will continue as heretofore and with the assistance of Dr. Clark the journal should become better and better.

Let the editors have your suggestions for features and articles which you would like to see next year. What are some of the practices and procedures that public schools are using in their modern programs in arithmetic? Are there serious questions that groups of teachers face and which might be presented and discussed in the journal? Help the editors to make this an interesting and worthwhile journal.

Big Dividends from Little Interviews

J. FRED WEAVER
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THERE IS NO "GET-RICH-QUICK" scheme for improving the effectiveness of arithmetic instruction, nor will there ever be such a scheme. There are available, however, sound low-cost investments which often yield substantial dividends at high rates of compound interest. These every teacher can afford, regardless of present salary status!

A Promising Investment

One of the most promising of these investments is an outgrowth of a marked change that has been occurring in arithmetic instruction during the past two decades: namely, our increased concern for the *process* of learning as contrasted with the *product* of learning. We have come to a fuller realization of the fact that *how* children learn is as important, and at times more important than *what* they learn. We have come to attach as much, and often more attention to the way in which children think as we do to the observable result of that thinking.

We know that all children do not think uniformly, in the same way, when dealing with a given quantitative situation. We recognize that important differences exist in the *levels* of thinking which children employ. Their thinking patterns range from those which are almost pathetically immature on the part of some of those which are rather startlingly mature on the part of others; from those which are stereotyped and inflexible to those which are insightful and ingenious.

We know, furthermore, that a given child does not tackle all quantitative problem-situations at the same level of thinking. There are differences for each child, just as there are differences among children, in the thinking patterns actually used.

Instruction in arithmetic cannot be most effective unless the teacher first is aware of the levels of thinking employed by the children in her class when dealing with various quantitative situations, and then differentiates her instruction appropriately in light of this knowledge. The teacher who seeks to become cognizant of these existing levels and who seeks to guide learning experiences accordingly, that teacher is the one who is making a promising investment—an investment that can and does yield substantial dividends, both present and future.

An Investment Technique

Various methods have been suggested and used to study children's thinking patterns in arithmetic. Buswell[1] discussed six of these in a significant article a few years ago. In the present paper the writer wishes to re-emphasize the importance of one of the techniques as a fruitful instructional procedure to be used by any classroom teacher. This is a form of *interview* in which children individually "think out loud" as they respond to specific quantitative situations, and related questions, which have been designed carefully for a specific purpose. The time spent in "interviewing" various pupils, and all children in a class upon occasion, will be well invested in terms of the dividends to be derived—the dividends of increased instructional effectiveness.

[1] G. T. Buswell, "Methods of Studying Pupils' Thinking in Arithmetic." *Arithmetic 1949*. Supplementary Educational Monographs No. 70. Chicago: University of Chicago Press, 1949, pp. 55–63.

Planning the Investment

Probably a specific illustration of the technique in actual operation may be advantageous. In this particular instance the classroom teacher, Miss Watkins, was anxious to become aware of the thinking patterns used by the pupils in her fourth-grade class as they responded to a group of multiplication combinations. She was ready to begin systematic instruction in multiplication and wanted to be guided to some extent by the children's existing skills and understandings relative to the basic facts of this process. She knew that during the previous year the pupils had worked with the facts involving 2, 3, and 4 as both multiplier and multiplicand. However, from past experience she also knew that there would be differences among the children in their level of mastery of these previously "taught" facts and in their level of understanding of multiplication as a mathematical process. Furthermore, she also knew that the same child would not necessarily respond to all combinations in the same way or at the same level.

Miss Watkins decided to use six representative multiplication combinations as the basis for her study of pupils' thinking patterns. Four of the six would involve supposedly "known" facts; i.e., facts that had been a part of the previous year's program of systematic instruction. Two of these four combinations would be presented in horizontal form and two in vertical form. Finally, Miss Watkins planned to include two "untaught" facts in the form of combinations having both multiplier and multiplicand greater than 4. These would not have been the object of specific teaching and practice in last year's systematic instructional program. One of the two "untaught" combinations would be presented in horizontal form and the other in vertical form. (The reason for including two facts of this nature, if not obvious to the reader, will be clarified shortly.)

Although Miss Watkins did not plan to measure understanding of the multiplication process directly, she felt that she could get some evidence of this important aspect of learning in either or both of two indirect ways. On the one hand, children who were unable to respond more or less "automatically" to one or more of the four "previously taught" combinations often would give an indication of their level of understanding by the method used to find the product. On the other hand, virtually no pupils would be likely to respond "automatically" to either of the two "untaught" combinations. Each of these would have to be "solved" in some way. Thus the method of attack and solution used by any child, in transferring his existing knowledge of the multiplication process and facts to these two new situations, generally would give some valid indication of his level of understanding of the process in question.

Implementing the Plan

Each of the representative combinations was placed on a separate 3″×5″ card as shown below.

$\boxed{\begin{array}{r} 7 \\ \times 3 \\ \hline \end{array}}$ $\boxed{\begin{array}{r} 5 \\ \times 6 \\ \hline \end{array}}$ $\boxed{2 \times 9 =}$

$\boxed{8 \times 4 =}$ $\boxed{\begin{array}{r} 9 \\ \times 5 \\ \hline \end{array}}$ $\boxed{7 \times 6 =}$

Miss Watkins presented these six cards, one at a time in the above order, to each of the children in her class. Whenever a pupil was unable to respond "automatically" to any combination, he "thought out loud" as he attempted to find the product. In these instances Miss Watkins would interject pertinent questions, etc, if she felt them to be necessary or helpful at any time. A class record sheet was employed to indicate the product given for each combination by every child and his method of response. When not "automatic," the thinking pattern used by the pupil was recorded as nearly verbatim as possible.

These "interviews" were not a new experience for the children. Consequently, Miss Watkins had no difficulty in establishing desirable rapport. The pupils knew what they were to do and responded freely and fully. They took the "interviews" seriously, and at the same time truly enjoyed them.

Such was not always the case, however. When Miss Watkins first tried the technique with these children in another type of situation, things did not run as smoothly! More time was needed, especially with some pupils, to establish rapport; to have them understand what they were to do; and to get them to respond naturally, freely, and fully. Nevertheless, with proper guidance and some degree of patience, it was not long before all children "caught on" and began to enjoy this type of experience thoroughly. Furthermore, Miss Watkins developed some short cuts and a scheme of coding which made the recording of responses much simpler and less time consuming for her.[2]

Studying the Investment Record

The pupils' responses and thought patterns were recorded in the form of a class chart to facilitate study of the data secured. An abbreviated illustration of Miss Watkins' completed chart is reproduced below. This shows the responses of

[2] Some teachers have found the use of a tape recorder to be distinctly advantageous, in that it frees them from having to make rather complete written records during the course of each "interview."

Illustrative Summary of Thought Patterns

Pupil	Previously "Taught" Facts		"Untaught" Fact
	$\begin{array}{r}7\\ \times 3\\ \hline\end{array}$	$8\times4=$	$\begin{array}{r}9\\ \times 5\\ \hline\end{array}$
Sally	Automatic Response Correct Product	Hesitated, then recited full "table": $1\times4=4$, $2\times4=8$, $3\times4=12$, etc. to $8\times4=32$.	Hesitated, saying "I don't know that." Then put down 5 rows of 9 dots and counted by ones to reach 45.
David	Automatic Response Correct Product	Said, "Let's see: 6 fours are 24, 7 fours are 28, 8 fours are 32. That's it."	Looked up a bit, then said, "Now I know how to work that one! 5 tens are 50, so take away 5 and that's 45."
Linda	Hesitated, then said: "7 and 7 are 14—15, 16—17, 18—19, 20—21."	Hesitated, then said: "Oh, I know! It's the same as 4 eights, and I remember that's 32."	Hesitated, saying "We never had that one before." Began to add 9 and 9; then suddenly she stopped and counted by fives to 45.
Carole	Automatic Response Correct Product	With seemingly no hesitation, said: "4 fours are 16; and 10, that's 26; and 6 more—30, 32."	Very little hesitation, then said: "Well, 4 nines are 36; and 10, that's 46; so take away 1, that's 45."
Jerry	Automatic Response Correct Product	Automatic response, but incorrect product of 36. When asked to "prove" it, looked puzzled and said, "I can't, but I'm sure that's what it is—36."	Looked very confused saying: "I don't remember *that* one. Did we have it before?" Was unable to attack it sensibly, asking: "Is it near 14?"

five children to three of the combinations —two of the "previously taught" group and one of the "untaught" group. Undoubtedly the reader will want to look over the chart at this point, and also will have occasion to refer to it from time to time during the discussion which follows.

An Overview

Even a cursory glance at the illustrative chart brings to light a distinct advantage of the interview technique. Miss Watkins more simply could have had each child copy the combinations on a sheet of paper and then write the products. Had she done this, the *observable result* likely would have been the same for Sally, David, Linda, and Carole: all four would have written the correct product for each combination. Miss Watkins easily might have drawn the erroneous conclusion that these four pupils had comparable mastery of the facts and understanding of the fundamental nature of the multiplication process, and that the next learning experiences could be virtually the same for each of the four children.

Such would have been far from the truth, however, Sally, David, Linda, and Carole actually differed significantly in various ways from the standpoint of the thinking patterns behind their ultimate overt responses. They differed in their level of mastery of the facts, in their level of understanding of the multiplication process, and in their level of ability to use other mathematical concepts and relationships. The same subsequent learning experiences would not be equally appropriate and effective for each of the four pupils. These facts were brought to light clearly through the interview technique but would have been obscured had Miss Watkins used a simpler alternative such as the one mentioned above.

Let us now look briefly at the record for each child listed on the illustrative class chart and direct attention to observations which deserve special comment.

Sally

Notice first that Sally used a common procedure in attacking the second fact when unable to respond automatically. She started at the beginning of the "four's table" and repeated the sequence of facts in order until she arrived at $8 \times 4 = 32$. Next observe that although Sally tackled the "untaught" combination at a relatively low level, she understood the meaning of the combination and proceeded accordingly. It is important to note that Miss Watkins could make no inference concerning Sally's understanding, however, from her response to either the first or second combination. The former response was automatic, and the latter might have come only from rote memory.

David

First observe that although David, like Sally, used the "four's table" to arrive at the second product, he did so in a more mature way. Rather than start at the beginning of the "four's table," David started with a "remembered" fact well along in the table ($6 \times 4 = 24$) and repeated the facts in sequential order from that point until he reached $8 \times 4 = 32$. Next notice the insightful method used by David to find the product for the "untaught" combination. Here is evidence not only of an understanding of the meaning of the combination, but also of a relatively mature attack based upon this understanding in relation to the distributive principle.[3]

Linda

This child had not reached the level of automatic response for either of the two "previously taught" facts. Each of these, as well as the "untaught" fact, had to be solved—but at different levels of thinking.

[3] This all-important principle is a *functional* one for many children even though they may not be able to express it concisely in the form of a verbal statement or mathematical equality: $a(b \pm c) = ab \pm ac$. In this specific instance, $5 \times 9 = 5\ (10-1) = 5 \times 10 - 5 \times 1 = 50 - 5 = 45$.

All methods of attack, however, gave evidence of understanding of one kind or another. Linda's approach to the first combination was at a relatively low, but not uncommon level. She used a "known" addition fact at the outset ($7+7=14$) and then resorted to partial counting. It is interesting to observe the rhythm employed in this latter connection.

Linda used the commutative principle[4] when dealing with both the second and third combinations, but at different levels. In the former instance she found the product for 8×4 by responding automatically to the "reverse" combination, 4×8. In the latter instance we see an interesting insightful procedure in which Linda applied this commutative principle in a way that enabled her to find the product much more easily than if she had continued to add nines as she started to do. It is significant to note that her attack upon the "untaught" combination was, in a sense, more mature (or at a higher level) than was her attack upon the first "previously taught" combinations.

Carole

The approaches used by Carole to the second and third combinations show effective use of the base of our number system (10) and its multiples (20, 30, 40, etc.), as well as functional application of other important mathematical relationships and understandings. In effect she recognized that 8×4 would be twice as much as 4×4, which she knew to be 16. Her method of doubling the 16 was far from stereotyped! In responding to the "untaught" combination, Carole worked from a related "known" combination (4×9) and then added 9 to the product by first adding 10 and then subtracting 1. In one way or another this child used more automatic responses to "known" facts than any of the other pupils did.

Jerry

This youngster's responses present an overall picture that is much different from any of the four which have been discussed previously. Nevertheless, the picture is far from unique or atypical—unfortunately! Multiplication as a mathematical process has very little meaning for Jerry, if any. He could respond automatically to "previously taught" combinations—sometimes correctly, sometimes incorrectly—but always mechanically to a meaningless stimulus. When questioned by Miss Watkins and asked to "prove" one of his products, he was unable to do so. He did sense that a "new" combination was involved at one point, but he had no way of coping with the situation. His ultimate response showed that he thought maybe he should add the 9 and the 5. Definitely, Jerry was in a class by himself when compared with the other four children.

Other Pupils

Obviously, the examples cited in the illustrative class record and just discussed do not embrace all of the thinking patterns, etc. revealed to Miss Watkins through her interviews. At times she needed to inject more questions of her own than she did in the illustrative instances. She found some difficulties, too, which were not in evidence for these five children. She observed that a few pupils were somewhat confused by the horizontal algorism and seemed uncertain about responding to combinations in this form. For one or two children just the opposite was true. Furthermore, she noticed that some pupils read the vertical algorisms downward rather than upward (e.g., "7 times 3" instead of "3 times 7"). Many of the patterns of thinking etc. were similar to those observed by Brownell and Carper[5]

[4] This useful principle asserts that if $a+b=c$, then $b\times a=c$. Although the interchange of factors does not affect the size of the product, it *does* alter the meaning of the situation. For example, 8×4 has the same product as 4×8, but the expression "8 fours" does not mean the same thing as the expression "4 eights."

[5] William A. Brownell and Doris V. Carper, *Learning the Multiplication Combinations.* Duke University Research Studies in Education, No. 7. Durham, N. C.: Duke University Press, 1943. 177 pp.

when interviewing children in connection with their excellent research study, *Learning the Multiplication Combinations*. Of course, some different patterns were observed as well.

All in all, Miss Watkins gained invaluable information about the strengths and weaknesses of her fourth-grade children in relation to basic facts and understandings of multiplication at the outset of her proposed program of systematic instruction in this phase of arithmetic content. Miss Watkins had made her investment, but where are her dividends?

Earning the Dividends

Miss Watkins, and all teachers, must beware. Highest dividends from interview-investments do *not come automatically*. They must be *earned*—earned by putting the investment to use. Only in that way can the attractive dividends of increased instructional effectiveness be realized most fully.

Put the investment to use? In what way? By planning and implementing differentiated instruction based upon information brought to light through a study of the investment record. As is true of so many things, often this may be "easier said than done." To indicate in detail how Miss Watkins used the data she gathered through the interviews—that would be another paper in itself. However, we must look briefly to one of the possibilities, to one of the things that might be done. We must catch a glimpse of one way in which the dividends could be earned.

A Promising Plan

Miss Watkins did *not* have her class organized formally in groups for arithmetic instruction in the same way that many teachers do. That is, she had not set up several groups (three, let us say) at the outset of the year's work and then provided mostly separate and independent instruction for each of the groups, permitting them to progress at their own rates—with all children in the class working together on common projects, etc. only rather infrequently.[6]

Although Miss Watkins did not organize her class formally in two or more groups and follow the instructional scheme just characterized, she did use groups many times in her teaching—but in a significantly different way. She employed a much more flexible plan of grouping—one that developed as the need arose, following her frequent exploratory work with the class as a whole in which a "grouping effect" generally prevailed. Miss Watkins had attempted to translate into action a promising plan advocated and illustrated by various persons,[7] including the present writer,[8] which places major emphasis upon differentiation in terms of *level* or *depth* of learning rather than rate of progress as we often conceive it in the frequent grouping procedure.

A Glimpse of the Plan in Action

When Miss Watkins began her reteaching of the understandings and skills of multiplication involving the basic facts with 2, 3, and 4 as multiplicand and multiplier,[9] her initial approach was through

[6] For a recent exposition of this point of view, see Charles E. Johnson's "Grouping Children for Arithmetic Instruction." The Arithmetic Teacher 1: 16–20; February 1954.

[7] See, for example:

John R. Clark, "A Promising Approach to Provision for Individual Differences in Arithmetic." *Journal of Education* 136: 94–96; December 1953.

John R. Clark and Laura K. Eads, "Teaching Children in Groups." *Guiding Arithmetic Learning.* Yonkers: World Book Co., 1954, pp. 245–252.

Rolland R. Smith, "Provisions for Individual Differences." *The Learning of Mathematics, Its Theory and Practice.* Twenty-First Yearbook, National Council of Teachers of Mathematics. Washington, D. C.: The Council, 1953, pp. 271–302.

[8] J. Fred Weaver, "Differentiated Instruction in Arithmetic: An Overview and a Promising Trend." *Education* 74: 300–305; January 1954.

[9] It is to be understood that the basic facts involving 1 and 0 were included. Products for such facts were considered in relation to the broad generalizations governing the use of these numbers as factors.

work with the class as a whole. The interview data were used to good advantage in this connection, giving her some idea of the strengths and weaknesses of each pupil and an indication of the level at which he might participate most successfully in the instructional activities.

Jerry and a few other youngsters definitely needed help in gaining and understanding of the nature of multiplication as a mathematical process. She knew that Sally and a number of other children would be able to interpret this meaning for Jerry and those like him at the concrete or semi-concrete levels. Still other pupils would be able to extend the interpretation to the more abstract levels, symbolically relating multiplication to addition, etc.

Thus, each child was enabled to contribute and profit at his own level of learning, and the interview data gave Miss Watkins a most helpful indication of the level at which she should encourage each pupil to work at this point of her reteaching. Actually, a "grouping effect" was in operation even though the children worked together as an entire class.

Frequently, however, situations arose which made it desirable for Miss Watkins actually to separate the class into several groups and work with each somewhat independently for a while. Often these groups would deal with the same broad aspect of multiplication, but at different levels.

For example, at one point in her reteaching Miss Watkins injected the idea of organizing related facts into a "table" for further study of relationships, etc. She had the entire class consider a situation such as the following:

Some boys in the school shop were making 4-wheeled toy wagons for Christmas presents to be sent to children's hospitals.

How many wheels would they need for 1 wagon?
How many wheels would they need for 6 wagons?
How many wheels would they need for 3 wagons?
How many wheels for 8 wagons?
How many wheels for 5 wagons?
How many wheels for 2 wagons?
For 9 wagons?
For 4 wagons?
For 7 wagons?

As the products were found and verified at different levels, they were recorded as follows:

$$\begin{array}{ccccccccc} 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 \\ \times 1 & \times 6 & \times 3 & \times 8 & \times 5 & \times 2 & \times 9 & \times 4 & \times 7 \\ \hline 4 & 24 & 12 & 32 & 20 & 8 & 36 & 16 & 28 \end{array}$$

Discussion of these recorded facts led pupils to rearrange or reorganize them in the manner indicated below:

$$\begin{array}{ccccccccc} 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 \\ \times 1 & \times 2 & \times 3 & \times 4 & \times 5 & \times 6 & \times 7 & \times 8 & \times 9 \\ \hline 4 & 8 & 12 & 16 & 20 & 24 & 28 & 32 & 36 \end{array}$$

Miss Watkins knew from her interview data, and from the work she had just done, that the children differed materially in the level at which they could deal with this type of situation and the depth to which they were able to perceive relationships among the facts.

Consequently Miss Watkins divided her class into separate groups—three, in this instance—characterized somewhat as follows:

Group 1—those who still needed to use representative materials frequently and who showed very limited perception of relationships among the facts of a table.
Group 2—those who generally could work at an abstract level and who had a somewhat better grasp of relationships among tabled facts.
Group 3—those who showed definite ability to deal with higher-level relationships in abstract form.

The first group reorganized sets of facts into systematic tables and dealt mainly with an understanding of the presence of the constant factor in a table and its relation to the increasing products. Representative materials were used frequently as needed.

The second group worked with this idea as well, but more quickly and without the use of concrete or semi-concrete materials.

Furthermore, they used their increased understandings to determine the next fact in a table following one given in isolation, and also used many "known" facts in this way to find the product for "unknown" combinations.

The third group dealt with the previous ideas only briefly, since these children already were rather secure and proficient in work of this nature. Most of their instruction centered around a deeper understanding of relationships inherent in the table of facts: recognizing that 6 *fours* would be twice as much as 3 *fours*, that 4 *fours* would be half as much as 8 *fours*, that 7 *fours* would be as much as 5 *fours* and 2 *fours*, or as much as 4 *fours* and 3 *fours*, etc.

Later, Miss Watkins again was working with the class as a whole. Still later, she was dealing with separate groups once more. In any event, levels of learning were constantly in her thinking as she planned and implemented her instructional program. Children were encouraged to work at the highest level of which they were capable and to move to higher levels as quickly as feasible. Consequently, a given child was not always placed in the same group when Miss Watkins felt the need to form such; nor was it impossible for a child to move from one group to another at virtually any time he showed evidence of working better at another level.

As her program of arithmetic instruction progressed throughout the year, Miss Watkins continued to use the interview procedure. At times she would study the thought patterns of only some of the children; at other times, of the entire class. In any event, she used the interview to gain helpful and truly *necessary* information—information that influenced her instruction greatly but which would have been obscured frequently without use of the technique.

Concluding Statement

Children's thinking patterns in arithmetic are highly important. Every teacher can make a sound investment by interviewing children in her class periodically to determine and study their levels of thinking when dealing with various quantitative situations. By then using the knowledge gained from such interviews to assist her in providing a helpfully differentiated program of teaching and learning experiences, every teacher can reap big dividends in the form of increased instructional effectiveness.

Editor's Note: Dr. Weaver has shown that different children think, discover, and respond by different methods which apparently show different levels of "maturity." One of the high arts of teaching is in knowing how to stimulate and lead pupils into more mature levels of performance. It is probably as wrong to try to teach a standardized method of discovery as it is to teach facts for rote memorization. The human mind is very complex and true learning is a growth process and not merely repeating the learning of others. The "flexible grouping" described by Mr. Weaver has many advantages which are social as well as mathematical. A teacher, like Miss Watkins, must through training and experience have gained a high level of discernment, she is to be prized.

A PERCENTAGE BOARD

Miss Inez Bailey of Rawlins, Wyoming likes to use a large square divided into 100 small squares so that each small square represents 1% and the large square 100% or a whole. When a per cent is mentioned, the children count as many squares as are necessary to represent the given per cent. They then find what part this is of the large square. This teaches them per cent and fractional equivalents in a meaningful way. When a part of a per cent is introduced, they again study their percentage square and see that only a part of one of the small squares is meant. This helps children to visualize and understand the difference between $\frac{3}{4}$%, for example, and $\frac{3}{4}$ or 75%. It is a simple but very helpful device.

Such a percentage board can be made of materials such as oilcloth, muslin, slated blackboard cloth, or any suitable material.

Flexibility in the Arithmetic Program

To Promote Maximum Pupil Growth

MAUDE COBURN
Oakland Public Schools

I HAVE HAD THE PRIVILEGE of observing arithmetic in many classrooms where arithmetic has real meaning for children. Would you like to visit with me in some of these modern classrooms where children really live their arithmetic?

First, we shall visit a primary room. As we enter, we see several children standing in the front of the room. Other children are at the chalkboard ready to record the number present and the number absent as each leader announces the result of the count of his group. As we watch and listen, we see and hear the class, with the help of the teacher, find the total number of children present and the total number absent. While this is being done, other children are counting those present by ones or twos to check the accuracy of the record on the board.

It is Friday, the day of the week on which milk money is collected. Milk costs thirty cents a week. Some children have brought three dimes, some have brought six nickles, other five nickels and five pennies. One has brought a quarter and a dime and announces that he needs a nickel change. Another child who needs experience in handling money is named to give the correct change. All the ways of making thirty cents are discussed. Dimes, nickels, and pennies are stacked, so the total amount of money can easily be counted.

It is time for reading. Twelve chairs have been counted and placed in a semicircle near the teacher's chair in the front of the room. Thirteen books are on the table, having been counted out by the monitor of the day. There are thirteen books because there are twelve children in the group, and the teacher makes one more, or thirteen. The teacher remarks that there are visitors in the room and inquires, "How many more books do we need so each visitor may have a book?" Then she proceeds, "Yesterday we finished our story on page. 15. On what page shall we begin today? Can you find that page?" She may not even have a chance to ask these questions before someone in the group will say, "Our story is on page 16 today." In this case, the teacher then asks, "How do you know?" There are many opportunities during the reading period to use numbers without disrupting the reading program.

Let us go on to a third grade. Here we find the class planning an excursion to the harbor. Eight mothers are going to take the children in their cars. There are thirty-six pupils in the class. How many children can go in each car? How long will it take to go to the harbor? What time will they have to leave the school in order to get there in time to take the boat which leaves at 9:30? How much money will all the tickets cost, if each fare is thirty-five cents?

You may say, "But these children in the third grade can't divide by eight. They can't multiply thirty-five cents by thirty-six." Don't be too sure until you have tried it. Children have ways of finding answers, if they are allowed freedom to think. They may not react on an adult level, but that is not necessary at this point. They are not yet ready to learn multiplication or division on an abstract level. What we are interested in now is whether they are doing some critical thinking, some real problem solving, some estimation, and whether they are acquiring an interest in and an appreciation of

the use of numbers in everyday living.

When we enter the sixth-grade room we have no trouble knowing what the interest unit is. Everything in the room clearly shows that aeronautics is taking first place. There are model planes built to scale. There is a time line around the top of the chalkboard showing the progress of airplane invention. Graphs and charts show comparisons of speed. Maps show air lanes. The boys and girls are in small discussion groups. One group may be studying air maps and comparing distances. Another group may be discussing the instrument panel they have constructed, while still another group may be discussing the newspaper article about the latest speed record of planes.

Learning Through Experience

Have we seen examples of real arithmetic in these rooms? Can you sense a change in the methods of teaching arithmetic? You may be thinking that this is the old "incidental theory" of teaching arithmetic, but it is far from that. This is planned use of arithmetic and is only a part of the total arithmetic program, but an important part, real live application. This change has taken place largely because of the many studies made of how children learn. Learning is being recognized as a change of behavior through actual experience.

Who is responsible for this change of behavior in the classroom? The teacher is the instigator and guide, and the pupil is the active participant or learner. The experiences provided must make sense to the child and must be important to him.

We have observed learning taking place as a result of rich experiences. The motivating factor was interest. For some children this interest was already there, for others it was just developing. The experience was satisfying needs that the children had at the moment. Each child was taking an active part, according to his ability. He was not being held to a rigid standard of arithmetic for the grade. No marks were being given and no rewards were bestowed, but each child had a feeling of accomplishment and a feeling of belonging to the whole group, and that feeling could be sensed by anyone entering the room.

If we were to visit these rooms during the regular arithmetic period, I am sure we would find the same atmosphere. The teacher would not be dictating every move the child should make. He would be providing opportunities for the pupil to develop independence. He would be encouraging initiative. The child would be discovering answers for himself. The child would be helping in the planning of his own activities. This behavior would have been the result of good planning over a considerable period of time. Change of behavior doesn't take place over night. Sometimes we get in too much of a hurry and do not let things develop in the best possible way. We begin to have fears and then we push for rapid results.

The Need for Flexibility

A *flexible environment* is apparent in each of these classrooms. You know that real arithmetic is being experienced. It is the child's room. The bulletin boards have, for the most part, been planned and arranged by the children. Many charts around the room are pupil-made. There are reference books of varying reading levels available to the children. There is a table with arithmetic games and manipulative materials on it. Measuring devices are much in evidence. Experiments are in process with diagrams and charts to show planning and progress. It is a learning laboratory.

The classroom environment is such that the curiosity of the child is aroused. He enters the room with enthusiasm. He eargerly anticipates what is going to happen and looks around for something new which the teacher may have brought to stimulate interest and curiosity.

The desks in this classroom are movable

and are arranged according to the needs of the moment. If rearrangment of seats is not possible, there should be a table around which a small group may gather for discussion and planning. Even with fixed seats, there are ways of getting into closely knit groups. Children working as a group must be seated close together, so conversation can be easily directed and planning can take place under the least distracting conditions. Any written directions for the group can be placed on the chalkboard near where the group is sitting. Children move into groups for reading, so why not for arithmetic?

The arithmetic *program* must be *flexible*. Teachers and parents know that children do not develop at the same rate, nor in the same way, nor at the same time. Because of this variation, there must be a differentiated program in each classroom to take care of the individual child. Even though we understand that learning is an individual matter, we realize that instruction must be group procedure. Within the small group, the individual learns from his own activities, from his own reactions to the situations in which he finds himself according to his own interests and needs and at the level of his own ability. Each child achieves his own goal.

Know the Child

The teacher must take the child where he is and proceed from there. This means that the teacher needs to know something of the child's background of experience, both in school and at home. He will need to know about the child's ability to learn, his interests, and something of his needs. He will want to become acquainted with the child as a whole child: his personality, his likes and dislikes, and his problems.

The teacher accomplishes these things in several ways. First, he becomes acquainted with the children by chatting with them informally before school when they are helping around the room. He listens as the children talk with each other in the room or on the playground. He observes the children during activity periods when they are constructing, doing dramatic play, or participating in art activities.

Second, the teacher may have an interview or conference with the child, at which time he encourages him to tell why he likes or dislikes arithmetic. The child will be able to discuss some of his difficulties. This is the teacher's opportunity to show the child that he takes an interest in him as an individual and that they are going to work together. The child will recognize the teacher as his friend and will feel secure as a person. The child who has exceptional ability will be discovered and, through these interviews, he will be encouraged to plan experiences in keeping with his special interests and ability.

Third, the teacher can get some information about children from records which previous teachers pass on to him.

Fourth, the teacher will give formal tests which will show the child's level of achievement and readiness.

The teacher now will put all of his information together and will plan for the whole class, but will be able to do a better job because of his knowledge of the individuals who make up that class. There will be a great range of ability, but instructional groups can be formed to take care of individual differences.

Flexible Grouping

The teacher's handling of *groups* must be *flexible*. Several ways of grouping are possible in any classroom. The subject of grouping is vast; therefore, only brief mention will be made of types and an example or two will be given.

1. The entire class may work as a group toward the solution of a common problem. The excursion mentioned in the first part of this paper is an example of this type of grouping. The aeronautics unit in the sixth grade is on a more advanced level. The class was divided into smaller groups to work on different areas of the same topic. They will all come together as a

class, and each small group will contribute its thinking to the whole.

2. Sometimes groups are formed according to social interests. Games represent this type of grouping. Friends can work and play together.

3. At times, there is a group of especially gifted children. The teacher may plan a long-term project with this group. They will advance at their own rate of speed, but the teacher will be there to guide them when necessary and to see that they are challenged. Money and how it affects our way of living and the history of clocks and time are types of enrichment units suitable for this group. Both of these topics require research. They also afford opportunities for excursions, planning and arranging exhibits, and other activities. Of course, such experience units as these may be a whole class project with reading and number activities planned to fit the various levels of ability in the class.

4. The most commonly discussed grouping, and one which is necessary, is that based on specific mathematical needs of children. As has been stated above, all children cannot be expected to be ready for the same work at the same time. Pupils may be in different stages of learning of any one process. Some may need more experience with concrete materials; some may be in the semi-concrete stage; and others may be at the abstract level when they are ready for concentrated drill and application of what they have learned. In this type of grouping, the teacher is recognizing all stages from the most immature to the most mature. In the same class, the range of ability and understanding may be so great that some pupils are in the counting stage, some in the addition and subtraction stage, and others ready for multiplication and division. This ability grouping permits continuous progress which is necessary because of the sequential nature of arithmetic.

There must be *flexibility* in the types of *materials* in each classroom. The slow learner cannot be expected to work in the same book as the fast learner. Supplementary books on various levels must be provided. Reference books must be available for both the pupil and the teacher. In order to adapt the instruction to individual differences, suitable instructional materials must be available. For every new learning, concrete materials, semi-concrete materials, and work sheets on the abstract level must be ready.

Drill or practice must be *flexible*. It must be varied. Children do not progress to higher levels of learning by practicing the same material over and over in the same situations. The child should practice what *he* needs. Needs will be different for different individuals. Drill can be varied by being oral part of the time and written part of the time. Games are good media for drill. Practice in real situations is the best form of drill, but drill on the abstract level must also be provided at the right time.

Assignments and *daily lessons* must be *flexible*. Merely assigning the next page in the arithmetic book is not teaching. Usually the new concept will be developed from an experience taken from the daily life of the children. Multi-sensory materials will be used for pupil discovery. After the children have made some generalizations from the use of these materials and from the relationships they have made, they will use the textbook to reinforce their learning. In this way quantitative terms which apply to the new concept will be familiar to the child and will have become part of his oral language vocabulary before he is asked to read them.

The daily lesson should be varied. There will be changes of activities during the arithmetic period. Some of these activities are group discussion, group activity, individual projects, group projects, individual practice, board work, use of films and other audio-visual materials. In this paper, it is not possible to give examples of planning to show how the teacher manages groups in the daily program.

A Broader View of Evaluation

The *testing program* must be *flexible.* Attitudes and meanings, as well as skill in computation and word problem solving, must be evaluated. Some techniques of evaluation are observation, individual interviews, class discussion, pupil reports, as well as formal paper and pencil tests on quantitative thinking.

If tests are to have any value for the teacher, they must be carefully diagnosed to find the general area in which performance is unsatisfactory, and also to find specific areas of difficulty. Small instructional groups are formed on the basis of needs shown in these tests. The basic purpose of evaluation is improvement of learning. It is a way of finding out what each child knows and what he doesn't know. Then development of new concepts or reteaching of those topics not completely understood is begun.

Evaluation is an integral part of the instructional program. It must be a continuous process. It must be done in terms of the objectives set forth in the curriculum guides. Test data must be considered in their relationship to the background, mentality, and interest of the individual. Testing is not an end-of-the-year technique in order to give a mark for work done.

The Teacher and the Course of Study

In order for flexibility to be possible, the *course of study* or *curriculum guide* must be *flexible.* Because of traditional grade standards, some teachers are reluctant to accept the modern trend of flexibility. They still teach the fifth grader from the fifth-grade book regardless of his ability and achievement. If the curriculum guides could be written showing progression from level to level instead of using grade labels, many more teachers would forget grade standards and would build from where the child is in his learning toward a higher level, basing the mathematics on the social development of the child. I am sure we would then have higher achievement in arithmetic.

Since teachers are helping more and more in the writing of curriculum guides, more experimenting is being done in the classrooms. When teachers' ideas and suggestions are accepted and incorporated in these guides, they feel a sense of responsibility for planning a good arithmetic program.

Teachers are encouraged to think of the course of study as a guide for them, a framework within which they are expected to be creative and resourceful. They should feel obligated to adhere to the basic philosophy, but should also feel free to use their own initiative in trying out new techniques and materials. They must adapt the use of this guide to their particular classes at any particular time. They must know that this guide has been based upon recent research and trends toward better teaching. It contains the objectives or outcomes in terms of attitudes, meanings, skills, and usage. Each teacher must have these objectives clearly in mind, if he is to effect changes in the behavior of children. He will have them in mind, if he has had a hand in developing them.

Teachers believe that children learn by doing and know that they themselves learn in the same way. They are willing to accept new ideas and try them out in order to have a better program.

Teachers need to know arithmetic content. They must have mathematical understanding, before they can develop understandings with children. They must also know something about the learning process and be able to apply what they know in the classroom.

Teachers need encouragement and guidance. They must feel they have the confidence and support of those directly responsible for the program. They may get this through direct help from supervisors and principals. They may get it in summer workshops set up in the city in which they

are teaching. They may get it from faculty meetings or small group meetings with teachers from neighboring schools where there is a chance for give and take of ideas.

Teachers can get information and can follow current trends by reading educational magazines, pamphlets, bulletins, and yearbooks published by such organizations as the National Council of Teachers of Mathematics, The Association for Supervision and Curriculum Development, or the National Society for the Study of Education. This material must be routed to them for specific purposes, not for general reading in most cases, as it might be overwhelming at first.

School people are always open to attack, often because the public doesn't know what is going on. Parents are eager to know what the "new" arithmetic program is all about. Demonstrations by children, articles in the local newspaper, or talks by supervisors, have helped to make parents aware of the value of the current practice of making arithmetic meaningful. This has resulted in several cases in Oakland in requests for parents' classes where they are taught in the same manner in which their children are being taught.

These group sessions afford an opportunity to get over to parents the idea of grouping for instructional purposes. They accept the fact that children do not learn at the same time, the same rate, nor in the same way. They do not want their children pushed too fast into something for which they are not ready. They also realize that some children need to go faster. Our responsibility as school people is to inform parents of our modern methods. When our program is explained to them and when children know the reasons for what they do and are happy and secure in their relationships with the school, and go home full of enthusiasm about what they are doing—then, and only then, can we be sure of whole-hearted cooperation.

Summary

1. Flexibility in the program does not mean complete freedom of practice or lack of planning for progressive development of skills. Because of the nature of this topic, only brief mention could be made of the mathematical phase of arithmetic.

2. Flexibility has to do with the method, not the content, of arithmetic.

3. Because we feel that children learn by using all the senses, the program must provide for experiences in which the senses can be fully used. This means multisensory materials must be available.

4. If interest is a motivating force in learning, the program must be flexible enough to make use of these interests.

5. Flexibility stresses the use of arithmetic in other areas of the school program—the social studies, science, and everyday living. It is a chance for democratic living. Because arithmetic is "everywhere and all the time," we must recognize its importance and have respect for it.

6. Because we believe in individual differences in children, the program must be flexible enough to provide for these differences. These differences include native ability, background, interest, and achievement.

7. Because we believe in individual differences of teachers, the program must be flexible enough to allow for their creative, resourceful thinking.

8. Testing is an integral part of instruction. Varied types of evaluation must be used and the results put to the best possible use in improving instruction.

9. Because research is being carried on continuously, we cannot accept all of what we are now doing as the end. We want progress. Teachers should be involved in this research. The program must be kept flexible to allow for teacher experimentation.

10. In order for teachers to do better work in the teaching of arithmetic and to enable them to use flexible methods, they

must know more about arithmetic. We are hoping for more and better training in this field.

11. We believe that more and better learning takes place in a flexible type of program and that it allows for the continuous, wholesome development of the total child.

12. We respect each child as a person and wish to give him the best possible chance for developing into a responsible, well-adjusted citizen, and this is possible only when he is allowed to help in planning his own program and is given a chance to do his own thinking.

EDITOR'S NOTE: Miss Coburn is Supervisor of Elementary Education and 7th and 8th grade Arithmetic in the Oakland public schools. The material for this article was originally prepared for the 1953 meeting of the National Council at Atlantic City. Miss Coburn has described very well an ideal toward which the teaching of arithmetic is progressing in many sections of the country. Such schools are not "play schools," they can and do achieve excellent results. In such a school the role of the teacher is most important and to carry this role successfully the teacher needs a great deal of knowledge of arithmetic and its uses and of children and their development. Many school people are worried about the implications of the current trend. They fear that pupils will not learn to add or to solve problems. But such is not necessary, all good schools provide for organized learning and for drill or practice when and where needed. The bright child should probably spend his extra time in achieving depth rather than rushing too far ahead in the mathematical sequence. What do teachers think of the ideal expressed by Miss Coburn?

SUMMER INSTITUTES

AT NORMAN, OKLAHOMA, June 6–10. *Institute for Arithmetic Teachers* held in conjunction with the *Summer Institute for Teachers of Mathematics* (June 6–17). For information write to Mr. F. Lee Hayden. The University of Oklahoma, Norman, Okla.

AT LOS ANGELES, CALIFORNIA, July 5–15. *The California Conference for Teachers of Mathematics.* Elementary and Secondary. For information write to Mr. Clifford Bell, University of California, Los Angeles, Calif.

AT NEW BRUNSWICK, NEW JERSEY, July 6–15. *The Third New Jersey Institute for Teachers of Mathematics.* Elementary and Secondary. For brochure write to The Director of the Summer Session, Rutgers University, New Brunswick, N. J.

AT ATHENS, OHIO, June 14–17. *Ninth Annual Conference in Elementary Education* featuring arithmetic. For additional information write to Dr. Holbert H. Hendrix, Ohio University, Athens, Ohio.

EDITOR'S NOTE: These summer institutes usually combine work with recreation in a short intensive session. They bring state and national leaders into informal work with teachers. College credit is usually available for those who complete the prescribed work. The workshop sessions "show how" newer ideas and materials are used in the classroom.

REVIEWS

The following pamphlets are "Educational Service Publications" available from The Extension Service, Iowa State Teachers College, Cedar Falls, Iowa. They are reviewed by Miss Angela Pace.

ISSUE NO. 1. *Arithmetic: Developing the Fraction Concept in the Lower Elementary Grades,* H. VanEngen, 1946. 10 pages, 10 cents.

Teachers who are concerned with effective procedures for developing and extending understanding of fractions in the first four grades will find many helpful suggestions in this bulletin. It shows how everyday objects and simply prepared materials can be used in developing the basic fraction concepts.

ISSUE NO. 20. *Solving Arithmetic Problems Mentally,* Jack V. Hall, 1954. 10 pages, 10 cents.

Within recent years there has been a trend toward renewed interest in the importance of mental arithmetic. This bulletin provides a wealth of material on this topic which should be of value to the teacher. The meaning, the values, and the teaching of mental arithmetic are discussed. The author's illustrations of procedures used by children in solving problems mentally at different grade levels are of special interest.

The Role of a Principal in Teaching Arithmetic

LAURA NEWELL
Principal, Jemison Elementary School, Tuscaloosa, Alabama

THE LIFE OF AN ELEMENTARY school principal is an interesting and challenging one. During the course of a day and all during the year, he receives many genuine satisfactions from his work, as well as sharing many interesting experiences. These experiences and satisfactions come through close contacts with teachers, pupils, and parents. It takes all these groups working together to have a good school whose program is responsive to the needs and interests of the children in the community.

All parents desire and feel that their children need instruction in the three R's. For many years the public schools have suffered many attacks about the teaching of these subjects. The teaching of arithmetic has definitely come in for its part of the criticism, especially during the past ten years. When boys went into military service, some knew very little mathematics so the attack was intensified. The colleges, high schools, and elementary schools started evaluating anew their program of instruction and realized that there was a definite need for understanding on the part of all so that a program of study might be planned which would attempt to meet the needs of the pupils.

The content of elementary school arithmetic has changed very little through the years, but methods of teaching this content to pupils are changing markedly. Some of these changes in method represent entirely new and different approaches to helping children understand and master arithmetic, while others are only slight variations or revisions of older methods. Many of our teachers are not aware of these changes and continue to follow procedures of telling, explaining written forms, giving assignments, hearing lessons, and checking papers, expecting the pupils only to do exactly what they are told.

Encouraging Faculty Study

The principal has an important job in helping acquaint his teachers with current trends in the teaching of arithmetic. Is is necessary that teachers be alert to improvements in teaching methods so that the instructional program will meet the needs and the abilities of the pupils. The teachers and principal working together through faculty-study groups are able to plan a program of study in arithmetic which will have more meaning for the pupils. If the principal is to be of service in formulating such a program, he must be aware of the trends and must have studied new approaches to determine their value and use in his situation. He must be sold on them in order to sell them to his teachers. He must be prepared to aid a planning committee of teachers as they select and secure materials to be used in their study. These materials would include professional books on the teaching of arithmetic, copies of various textbooks which use new and different approaches, monographs and magazine articles pertinent to the subject, as well as samples of aids to be used in the teaching of arithmetic.

After the materials collected have been studied, discussed, and evaluated, the teachers begin to realize that the task of the arithmetic teacher involves the creation of situations in which children actively seek answers to their questions. The teaching plans, the organization of the classroom, the interest and enthusiasm of the teacher will do much to stimulate pupil growth. This organization will provide opportunities for pupils to explore and question, to use their own ingenuity in making experiments and in trying various ways of doing things so that they may arrive at the best solution. In such

an arrangment the teacher and the pupils will be alert to classroom situations which present meaningful experiences for pupils in arithmetic. Teachers must realize that one must be patient, must go only as fast as pupils can go in learning, must use many concrete and semi-concrete aids in teaching all phases of the arithmetic program, and that these may be used on all grade levels.

Securing Concrete Aids

Another responsibility of the principal is helping teachers secure concrete aids to be used in the teaching program. Many teachers realize the value of the use of such aids; but, since funds to purchase them are limited, they have very few to use in the classroom. Many aids are very inexpensive and easily made. The children love to help collect such aids as spools, popsicle sticks, milk bottle tops, dominos, beads, tickets, and other similar items to be used in counting and in proving number combinations. The abacus, peg boards, tens blocks, number charts, and flannel boards may be made from plywood pieces of scraps. Most school communities have a wood working shop of some kind where the sawing or drilling may be done. Teachers and students in high school shops and trade schools are willing to help if approached. When parent groups become interested, many aids may be made. The children welcome opportunities to help in sandpapering, varnishing, or painting aids. Costs are reduced to a minimum.

Acoustical tile used with golf tees makes an excellent aid in teaching arithmetic. Contractors are glad to give pieces that are soiled or broken to the schools. The golf tees may be purchased for a nickel a dozen. This makes a very inexpensive and useful aid for teaching almost any process in arithmetic. The golf tees can easily be inserted to show various number fact groupings, decimal system patterns such as 10 tens in a square arrangement, or other organizations of number. Or three golf tees and a length of string can quickly be arranged to outline a neat triangle. Needless to say, the use of the above aids would enrich the program of teaching and results would soon be apparent. As teachers use aids they discover new uses and new aids each day.

Stimulating Experimentation

Most teachers need encouragement to experiment and try out new ideas. The principal through his interest and enthusiasm is able to encourage and stimulate teachers who are otherwise hesitant to try new methods and new approaches. The needs of pupils—average, gifted, and slow-learning—are better met through participation in small groups within the large group. The materials used by each group vary and new materials which the pupils have not seen or used before must sometimes be secured. Here again the principal can definitely help the teachers, especially the new teacher. She usually needs much help in organizing her day, setting up her program, and collecting materials. She looks to the principal for guidance. This is another reason why he must be alert to new methods, ideas, and ways of teaching so that he may help her get off to a successful start.

Evaluating the Teaching

After teaching comes evaluation. The principal has a definite responsibility in helping teachers evaluate the practices and methods used to see if they are fulfilling the objectives set up in the beginning, that of meeting the needs of the children of the community. The measures used for evaluation are set up for each particular situation and should vary from group to group. The final measure of the value of the whole faculty study is the development of a better understanding of arithmetic by teachers and pupils alike.

All in all, the job of the elementary principal in the instructional program is one of cooperation with the teachers by making himself available to advise, to

(Concluded on page 59)

Addition, Subtraction, and the Number Base[1]

CLIFFORD BELL
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WHEN THE TEN ONE-DIGIT NUMBERS are added two at a time in all possible ways, we obtain the 100 addition facts as shown in the following table.

PRIMARY ADDITION FACTS

0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0
0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8	9	10
0	1	2	3	4	5	6	7	8	9
2	2	2	2	2	2	2	2	2	2
2	3	4	5	6	7	8	9	10	11
0	1	2	3	4	5	6	7	8	9
3	3	3	3	3	3	3	3	3	3
3	4	5	6	7	8	9	10	11	12
0	1	2	3	4	5	6	7	8	9
4	4	4	4	4	4	4	4	4	4
4	5	6	7	8	9	10	11	12	13
0	1	2	3	4	5	6	7	8	9
5	5	5	5	5	5	5	5	5	5
5	6	7	8	9	10	11	12	13	14
0	1	2	3	4	5	6	7	8	9
6	6	6	6	6	6	6	6	6	6
6	7	8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7	8	9
7	7	7	7	7	7	7	7	7	7
7	8	9	10	11	12	13	14	15	16
0	1	2	3	4	5	6	7	8	9
8	8	8	8	8	8	8	8	8	8
8	9	10	11	12	13	14	15	16	17
0	1	2	3	4	5	6	7	8	9
9	9	9	9	9	9	9	9	9	9
9	10	11	12	13	14	15	16	17	18

A great number of teachers will point out early in the teaching of these addition facts that a fact and its reverse gives the same sum. Thus $3+4=7$ and $4+3=7$. Hence all the facts in the table below the broken double line can be considered the same as their reverses which are all found above the double broken line. Thus, the omission of these reverses leaves only 55 facts to memorize. The zero addition facts in the first row will entail no memory work since teachers usually can convince a child that the sum of any number and zero is the number. Hence the top row of facts, above the single line, may be deleted, leaving 45 facts, or possibly 46 if we count the statment about adding zero as one fact. These are the addition facts that the children usually are required to memorize.

Many teachers have divided the aforementioned 45 facts into easy, average, and hard groups in accordance with the results of experimentation with children. None of the 25 facts whose sums are 10 or less are classified in the hard group. (These are bounded in the table by the broken double line to the left, the broken single line to the right, and by the single line at the top.) It is my belief that these facts should be taught first followed by the remaining 20 facts whose sums are greater than 10.

Since present day elementary teachers make a great deal of use of the idea of grouping objects into units, tens, hundreds, etc., they will find this an excellent approach to the teaching of these last 20 addition facts. For example, take the fact

[1] Adaptation of a talk given at the Seattle Meeting of the National Council of Teachers of Mathematics, August 1954.

$7+5=12$. The 7 may be represented by 7 sticks and the 5 by 5 sticks. These can be regrouped into two groups such that one of the groups contains 10 sticks. We need to known how many sticks must be added to 7 sticks to give us a group of 10 sticks. From amongst the first 25 addition facts, we find the answer to this question for $7+3=10$. Now we have one group of 10 sticks and a second group of only 2 sticks. Making use of our number base 10 and the place system of notation, we can write the sum as one 10 and two 1's or simply 12, the required sum. This method has been tried in the classroom,[2] and many teachers are now using this procedure to teach addition.

This method of learning the primary addition facts with sums greater than ten, puts a great deal of emphasis on the facts $9+1=10$, $8+2=10$, $7+3=10$, $6+4=10$, $5+5=10$. Two numbers whose sum is 10 are said to be complementary and each number is said to be the complement of the other. Thus, 7 is the complement of 3 and 3 is the complement of 7. Some teachers may object to defining this term since it may confuse the pupil. However, it may have just the opposite effect, since the definition tends to set these facts apart and thus their learning may become easier.

As is well-known, from each addition fact for which the addends are different, two subtraction facts are obtained. For example, $5+2=7$ gives the subtraction facts $7-2=5$ and $7-5=2$. For addition facts with the same addends only one subtraction fact is obtained. Thus $4+4=8$ gives only $8-4=4$. Hence the 25 addition facts whose sums are 10 or less give rise to 45 subtraction facts, since there are 20 with different addends and 5 with the same addends. These are the subtraction facts which should be memorized.

[2] At the Seattle Meeting of the National Council of Teachers of Mathematics, Miss Dora Brosius, Lakewood Schools, Lakewood, Oregon, gave an interesting report on her success with the use of this method.

Developing Subtraction

Actually in the regrouping method of addition, the ideas of subtraction are easily introduced. Thus, in the example $7+5=12$, when the child regroups to form a group of 10, he is discovering what he must add to 7 to make 10. This is the additive method of performing the subtraction $10-7=3$. Now when he takes the 3 from the other group, he will find how many are left at first by counting. Nevertheless he is performing the subtraction example of 5 take away 3, which is commonly called the take-away method of subtraction. Thus by the time the child has mastered his addition facts, he has actually learned the meaning of subtraction. With a little well-directed help from the teacher, memorization of the 45 simple subtraction facts will be easy for the child.

The remaining primary subtraction facts are those which are obtained from the addition facts in the table bounded below by the broken double line and above by the broken single line. The reader may readily verify the statement that there are 36 such subtraction facts, and these are the ones that usually cause trouble. Again by the proper utilization of the number base, these can be made easy. Let us consider $15-7=8$. In teaching this fact, the 15 may be regrouped into the number base group of 10 objects and the group of 5 objects. The teacher should now point out to the child that if he wishes to take seven objects away, it would be best to take them from the larger group of objects. The child then thinks of his problem as 10 take away 7. This fact has already been learned, since it occurred amongst the easy subtraction facts. If the teacher has used the term "complement," the answer to $10-7$ is the complement of 7, which is 3. Since these 3 objects together with the other group of 5 objects gives 8 objects, the fact $15-7=8$ is established. All of these 36 harder subtraction facts may be obtained in a similar

manner and when presented this way most children grasp these facts very easily. Actually this method eliminates the tedious memory work for these harder subtraction facts.

This method, in which the number base plays an important role, may be used to advantage in other subtraction examples such as $54-18=?$ Writing this in vertical form and supplying the necessary crutches for the benefit of the reader, we have

$$\begin{array}{rr} {}^{4}\cancel{5} & {}^{10}4 \\ -1 & 8 \\ \hline 3 & 6 \end{array}$$

Since 8 can not be taken from 4, we must regroup the 5 tens indicated by the 5 in the left hand column into 4 tens and 1 ten. This is indicated by the small 4 over the 5 in the left column and the small 10 over the 4 in the right column. Now the procedure is to say, "10 take away 8 is 2 (which is the complement of 8); 2 and 4 are 6." In the next column we simply have 4 take away 1 which is 3. Of course, the crutches would be abandoned after understanding of the method is established.

This method is not new, but the method of presentation may be. It is known as the complementary method of subtraction and was used in more or less rule form extensively during colonial days in this country. As was customary in those days, no attempt was made to give understanding to the method presented. Gradually through the years, as it became apparent that better results were obtained when understanding of the operations of arithmetic accompanied the learning of facts, new methods of performing operations were introduced which seemed to give better understanding. The old complementary method of subtraction was abandoned and forgotten. Yet with our present-day teaching aids and emphasis on the number base, this method can be taught with as much, or even more, understanding than those in common use. It is hoped that the above discussion may encourage more teachers to try the methods of addition and subtraction indicated in this discussion.

Editor's Note: Readers who are familiar with old books will recognize the "complementary method." Is Professor Bell correct when he says that this method can be taught with as much, or even more, understanding than those in common use? Who will give the new-old method a fair trial? How do readers like the two-step "thinking-talk" used with two-digit subtraction? Is this a good final method or is it better as an intermediate step in thinking? The editor objects slightly to Professor Bell's use of the word *memorized* in passages such as "... facts which should be memorized." The word *learned* is preferred because so many people have come to associate memorizing with a method of learning which is often called "rote memorization." It is clear that the author does not wish to imply rote memory because he used objects and understanding based upon the nature of the number system.

ROLE OF A PRINCIPAL

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lend a helping hand, to help secure needed materials, and to encourage the teachers in their efforts to improve teaching procedures. The principal and teachers, by all working together, are able to create a learning atmosphere in which arithmetic is taught with increased meaning. Children participate in experiences which help them to develop an understanding of our decimal number system. They learn to compute, but they are not pressed to do computation before they are ready to do so throughtfully. Needless to say, the principal who helps create such a situation is helping the children in his school to learn more arithmetic, to learn it more thoroughly, and to enjoy his learning.

Editor's Note: Miss Newell has given a fine picture of the role of a school principal in facilitating learning. The spirit and atmosphere of helping, encouraging, and stimulating are very different from the older point of view so often characterized by "I am the boss" as an attitude. Teachers have a responsibility in "educating" principals who may seem dogmatic because of their own insecurity. In many states the amount of professional training required of a principal is comparatively small.

They Love Arithmetic!

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ARE CHILDREN MORE INTELLIGENT today than they were years ago? Perhaps not, but teaching methods have changed drastically and teachers today are able to recognize a pupil's ability, bring it to the fore, and help develop it in ways that make even a class in arithmetic fun. I say *even* arithmetic because this subject seems to frighten so many students in the elementary school, and this intrinsic fear is often carried into high school and college.

Contests, as such, are seldom used as a part of our arithmetic curriculum. We all know that the purpose of arithmetic today is not to achieve proficiency through constant drill, but to develop in our pupils an understanding of number and of its relationship to their daily living. Modern pedagogy rightfully permits the needs of the pupils in the classroom to determine the means by which a subject might best be taught. Strangely enough, in some of my sixth and seventh grade classes a few years ago at Burris Laboratory School of Ball State Teachers College, students requested contests, and I used them as a means of motivation. It is not implied in this article that the teacher supplant the usual topics in an arithmetic class by similar contests. However, as an instrument of motivation they can be effective, developing enthusiasm for arithmetic in a classroom where indifference or dread of the subject might otherwise exist. I learned one thing myself: never to underestimate the ability of the students in your classroom. I was astounded at the scope of understanding and the enthusiastic reaction of youngsters in these classes.

One day at the beginning of the school year, several of my students approached me and requested that I permit them to have "races." Not having the vaguest notion as to what constituted such a race, but remembering that ciphering matches were once popular in the schools, I thought that I might pursue this request and through it find an avenue to increase their enthusiasm for arithmetic itself while improving their manipulative skills. I did not desire to put too much emphasis on drill as such; however, I did desire to aid their self confidence while increasing their proficiency, and I sought to use these contests as a limited means. It was decided to deviate considerably from the old concept of ciphering matches and show the youngsters some of the alternate methods of performing the fundamental operations. These alternate methods are well known to teachers everywhere. They include the *scratch method* of addition and multiplication; the *additive process* in subtraction; *duplation*, *Gelosia* or *grating method*, and *Cross Multiplication* (restricted to two digit numbers) in multiplication; and the *French* or *Austrian method* in division.

We set aside one day each week, usually Friday, as a day to conduct the contest. The class of thirty persons was divided into five teams. Each person selected at random served as captain at least once. The captains selected their teams, one at a time, preferential appointment being determined by lot. It so happened that the more talented students were selected first and the slow learners generally last. As they competed against each other in the order in which they were chosen, we had competition between those of similar abilities and aptitudes rather than pitting a quick student against one less skilled in manipulation. I realized it would be unfortunate for me to attempt to group students according to their manipulative skill and achievements within the group. By the "democratic

process" of having the students themselves select their own team members, this necessary division was made without any feeling of resentment or inferiority on the part of the slower students. These slower students were given the opportunity to serve as captains in due time. Some of these students improved their proficiency to such a great extent that they were among the first selected as the contests continued.

Since some method of scoring the contest had to be designated, the class decided that they would make and keep bar graphs. Every member of the class kept score. Graphs were checked periodically by the teacher, and the students assisted each other in keeping correct scores.

As this phase of classroom work was initiated through the general interests of the class as a whole, it seemed desirable to allow one member of the class to lead the contest and arbitrate any disputes that might arise. The contest leader was elected by the class members to serve for only one contest, a new leader being selected the following week. Some of the youngsters proved themselves surprisingly capable in this capacity. In the case of a serious dispute, the teacher would make suggestions.

The class would vote on the different methods—usually not more than three—that they would use during the period. The contest leader would carefully review the rules and state which method would come first. A representative from each team would go to the board in consecutive order (the captains first, etc.). The leader would request some number (one, two or three digits) from a volunteer seated in the classroom. In the event that another number was necessary in the operation, the leader would request another number from still another volunteer. Those participating at the board would be permitted to put those numbers down only in a certain order, conforming to rigid rules which the class members themselves had made. They would turn their backs to the board and wait for the leader to give the signal to start. Once the signal was given, they would turn and attempt to carry out the new method first and check by the conventional, or common, algorism. If both answers checked, that student would exclaim: "Finished!" All the other contestants would stop at that time. Should some class member challenge the student, thinking there had been some infraction of the rules, and prove his or her assertion, the team represented by that contestant would not gain a point but would lose a point. This eliminated the possibility of one of the contestants announcing that he had finished merely to prevent another student from winning that particular contest.

These contests seemed to instill confidence in the children, increase their proficiency in the arithmetic skills, give them an opportunity for leadership, and provide an atmosphere of good sportsmanship that was carried into other classrooms and extracurricular activities.

Many teachers may be reluctant to inaugurate these contests for the simple reason that they may think confusion would arise by teaching the students these various methods of performing the fundamental operations. By insisting on a check of the procedure by the common, or accepted, method each time a problem was worked under the alternate method, pupils improved their manipulative skills considerably and no confusion was evident.

Since no control group was used in this project, it is difficult to ascertain whether any appreciable change occurred in the students' behavior beyond the normal growth. However, it seemed evident to me in the classroom and from remarks made to me by many of the parents, that the enthusiasm of the students for arithmetic had increased immeasurably along with their skills and self confidence. These were *their* contests; the students had requested them, had made the rules, and had gained enjoyment from conducting them. They answered a particular need in these classrooms, and served the teacher well in this instance, as a means of motivation.